

U.S. DEPARTMENT OF COMMERCE
National Technical Information Service

AD-A028 213

EFFECT OF SECOND ROTATION ON FREQUENCY-TEMPERATURE
CHARACTERISTICS OF AT-CUT CRYSTALS

ARMY ELECTRONICS COMMAND

AUGUST 1976



230117

ADA 028213

Research and Development Technical Report

ECOM -4424

EFFECT OF SECOND ROTATION ON FREQUENCY-TEMPERATURE
CHARACTERISTICS OF AT-CUT CRYSTALS

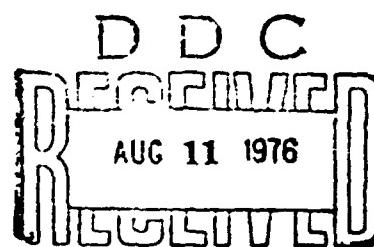
Arthur Ballato

Electronics Technology & Devices Laboratory

August 1976

DISTRIBUTION STATEMENT

Approved for public release;
distribution unlimited.



ECOM

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U. S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA 22161

US ARMY ELECTRONICS COMMAND FORT MONMOUTH, NEW JERSEY 07703

NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The citation of trade names and names of manufacturers in this report is not to be construed as official Government endorsement or approval of commercial products or services referenced herein.

Disposition

Destroy this report when it is no longer needed. Do not return it to the originator.

ACCESSION FOR	
NTIS	White Section <input checked="" type="checkbox"/>
DCC	Buff Section <input type="checkbox"/>
UNARMEDIZED	<input type="checkbox"/>
JUSTIFICATION
BY	
DISTRIBUTION/AVAILABILITY CODES	
GIST
A <input type="checkbox"/> B <input type="checkbox"/> C <input type="checkbox"/> D <input type="checkbox"/> E <input type="checkbox"/> F <input type="checkbox"/> G <input type="checkbox"/> H <input type="checkbox"/> I <input type="checkbox"/> J <input type="checkbox"/> K <input type="checkbox"/> L <input type="checkbox"/> M <input type="checkbox"/> N <input type="checkbox"/> O <input type="checkbox"/> P <input type="checkbox"/> Q <input type="checkbox"/> R <input type="checkbox"/> S <input type="checkbox"/> T <input type="checkbox"/> U <input type="checkbox"/> V <input type="checkbox"/> W <input type="checkbox"/> X <input type="checkbox"/> Y <input type="checkbox"/> Z <input type="checkbox"/> SPECIAL	

//

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ECOM-4424	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EFFECT OF SECOND ROTATION ON FREQUENCY- TEMPERATURE CHARACTERISTICS OF AT-CUT CRYSTALS	5. TYPE OF REPORT & PERIOD COVERED Technical Report - June 76	
7. AUTHOR(s) Arthur Ballato	6. PERFORMING ORG. REPORT NUMBER	
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Electronics Command ATTN: DRSEL-TL-ML Fort Monmouth, NJ 07703	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1T1 61101 A 91A 09 455	
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Electronics Command ATTN: DRSEL-TL-ML Fort Monmouth, NJ 07703	12. REPORT DATE August 1976	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES 15	
16. DISTRIBUTION STATEMENT (of this Report) Statement A. Approved for public release; distribution unlimited.	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE UNCLASSIFIED	
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Piezoelectric vibrators Piezoelectric crystals Transducers Crystal resonators Crystal preparation	Crystal cutting Crystal oscillators Frequency control X-ray orientation	
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) AT-cut quartz plates, the most widely used vibrators for frequency selection and control, are singly rotated cuts. Increasingly stringent requirements on frequency tolerances motivate this report, which examines the effect on frequency-temperature behavior of small but nonnegligible second rotations that arise from inevitable manufacturing variations. The results presented permit formulation of practical and consistent specifications of tolerances on both angles to achieve a required temperature coefficient.		

CONTENTS

	<u>Page</u>
INTRODUCTION	1
FREQUENCY-TEMPERATURE BEHAVIOR	3
ANGULAR VARIATIONS IN φ AND θ	4
CONCLUSIONS	8
REFERENCES	9
FIGURES	
1. Singly and Doubly Rotated Crystal Cuts.	2
2. Frequency-temperature-angle Characteristics of AT-cut Quartz Resonators.	5
3. Locus of Constant First-order Temperature Coefficient Shift, as Function of Angle Deviations.	7
TABLES	
1. Temperature Coefficients of Frequency for the AT-cut at Its Reference Angle.	3
2. Angle Derivatives of the First-order Temperature Coefficient of Frequency.	6
3. Angle Deviations Yielding Zero Temperature Coefficient Shift.	8

INTRODUCTION

Modern high precision frequency control, selection, and timekeeping applications impose extremely tight tolerances on the frequency-temperature behavior of crystal resonators.¹⁻⁶ These applications make it necessary to carry out systematic investigations into a number of effects once considered too small to warrant attention. Some of these effects include the influence of mass-loading on resonator frequency⁷⁻¹¹ and on temperature coefficient of frequency¹²⁻¹⁵ as well as the effect on temperature coefficient of harmonic^{6,12} and of operation at either resonance or antiresonance.^{17,12-15}

The effect of incremental angle changes has also been investigated, primarily by Bechmann.¹⁸⁻²⁰ His results treat variations in a single angle only and are limited to first order changes. However, they were adequate for many years until the advent of the most recent requirements.¹⁻⁶ With a view toward meeting these requirements, a search along the lines of earlier work²⁰ was made to determine the sensitivities of resonator design parameters and characteristics to angular changes.²¹ An outgrowth of that study was a determination of angle sensitivities in the vicinity of axes of symmetry; these results apply directly to that most widely used crystal plate--the AT-cut. This report describes how angle variations from the prescribed orientation of the AT-cut affect the frequency-temperature behavior.

Crystal plate orientation is described most generally in terms of three angles. For thickness modes, the direction perpendicular to the plate thickness is of no consequence, and orientation is specified by two angles, φ and θ , as shown in Figure 1. The standard notation for a doubly rotated cut is²² $(YXw1)\varphi/\theta$. The AT-cut is a member of the rotated-Y-cut family, and as such it is a singly rotated cut specified by a single angle θ , with $\theta \approx +35.25^\circ$. Its orientation may be specified as $(YXw1)0^\circ/\theta \approx +35.25^\circ$ or as simply $(YX1)\theta \approx +35.25^\circ$.

The AT-cut plate, with $\varphi = 0$, contains the diagonal symmetry axis X. This feature requires the vanishing of the elastic constants

$$c_{25}^E, c_{26}^E, c_{35}^E, c_{36}^E, c_{45}^E, \text{ and } c_{46}^E,$$

and the vanishing of the piezoelectric constants

$$e_{22}, e_{23}, e_{24}, e_{32}, e_{33}, \text{ and } e_{34},$$

with the consequent results that the desired mode (the slow shear mode) is a pure mode and that most of the important quantities have a zero φ derivative, so that departures in φ are only manifested in second order.

Variations in φ , about the value zero, arise invariably as a result of manufacturing processes; and the effects, particularly on the temperature coefficient of frequency, are no longer negligible and are described in this report.

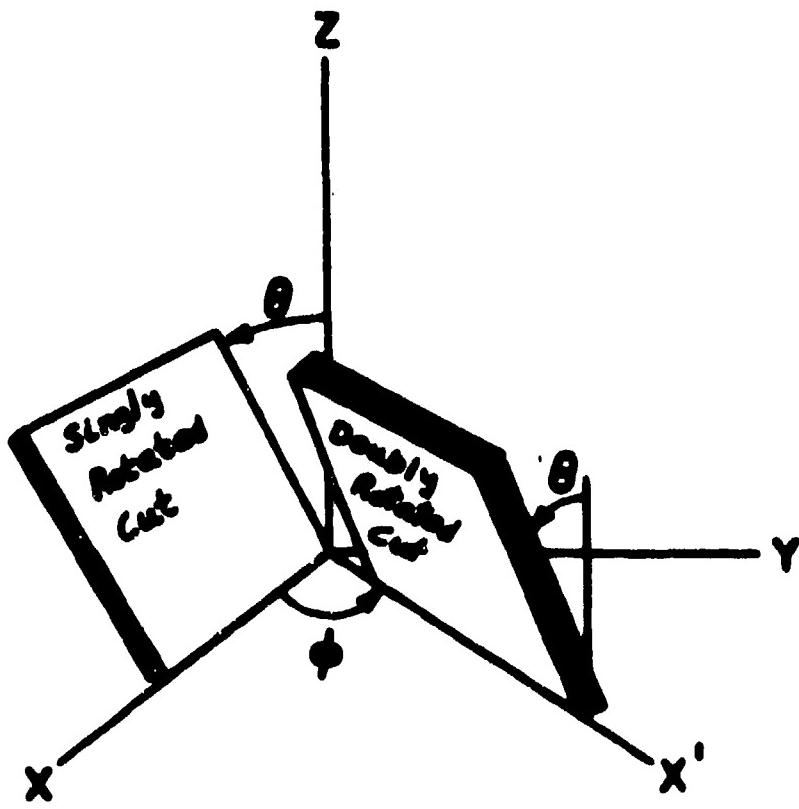


Figure 1. Singly and Doubly Rotated Crystal Cuts.

FREQUENCY-TEMPERATURE BEHAVIOR

Following Beckmann,^{18,19} we make a power series expansion about reference temperature T_0 , of the vibrator frequency:

$$f = f_0 + (\partial f_0 / \partial T)(T-T_0) + \frac{1}{2}(\partial^2 f_0 / \partial T^2)(T-T_0)^2 + \\ \frac{1}{6}(\partial^3 f_0 / \partial T^3)(T-T_0)^3 + \dots, \quad (1)$$

where $\frac{\partial^n f_0}{\partial T^n}$ means $(\partial^n f_0 / \partial T^n)_f = f_0$. For almost all applications it is sufficient to stop with $n = 3$. With the abbreviations $\Delta f = f-f_0$,

$$\Delta T = T-T_0, \text{ and } T_f^{(n)} = \frac{1}{n! f_0} \frac{\partial^n f_0}{\partial T^n}, \text{ we have}$$

$$\frac{\Delta f}{f_0} = T_f^{(1)} \cdot \Delta T + T_f^{(2)} \cdot \Delta T^2 + T_f^{(3)} \cdot \Delta T^3,$$

or as is usually written

$$\frac{\Delta f}{f_0} = a_0 \Delta T + b_0 \Delta T^2 + c_0 \Delta T^3. \quad (2)$$

The experimental values for a_0 , b_0 , and c_0 , and the inflection temperature

$$T_i - T_0 = -b_0/3c_0, \quad (3)$$

are given in Table 1, where $T_0 = 25^\circ\text{C}$, and f_0 is taken as the antiresonance frequency in the absence of mass-loading, f_{A0} .

TABLE 1. TEMPERATURE COEFFICIENTS OF FREQUENCY FOR THE AT-CUT AT ITS REFERENCE ANGLE.

θ_0	φ_0	a_0	b_0	c_0	T_i
degrees	degrees	$10^{-6}/\text{K}$	$10^{-9}/\text{K}^2$	$10^{-12}/\text{K}^3$	°Celsius
+35.25	0	0	-0.45	+108.6	26.4

Using the values of a_0 , b_0 , and c_0 from Table 1 in (2) generates the curve of $\Delta f/f_0$ versus ΔT shown in Figure 2 for $\Delta\theta = 0$. The angle deviation $\Delta\theta$ equals $(\theta - \theta_0)$, where θ_0 is usually taken as $\pm 35.25^\circ$, but will vary somewhat depending on mass-loading, harmonic, etc.⁷⁻¹⁷ Departures from the reference angle have been shown to produce experimentally the family of curves in Figure 2. This angular variation has been characterized by making a Taylor series expansion in $\Delta\theta$, and retaining one term each in $\Delta\theta$ for a_0 , b_0 , and c_0 .^{18,19}

In order to incorporate the effect of variations in φ , as well as θ , a generalization of the Taylor series approach is described in the following section and applied to the AT-cut.

ANGULAR VARIATIONS IN φ AND θ

The Taylor series in two variables can be written

$$F(x+A, y+B) = F(x, y) + (\partial A/\partial x + B \partial/\partial y) F(x, y) + \dots + (\partial^2 A/\partial x^2 + B \partial^2/\partial y^2)^n F(x, y)/n! + \dots . \quad (4)$$

Letting $F = T_f^{(1)}$; $x, y = \theta, \varphi$; $A, B = \Delta\theta, \Delta\varphi$, and retaining terms up to second order gives

$$a = a_0 + (\partial a_0/\partial \theta) \cdot \Delta\theta + (\partial a_0/\partial \varphi) \cdot \Delta\varphi + \frac{1}{2}(\partial^2 a_0/\partial \theta^2) \cdot \Delta\theta^2 + (\partial^2 a_0/\partial \theta \partial \varphi) \cdot \Delta\theta \Delta\varphi + \frac{1}{2}(\partial^2 a_0/\partial \varphi^2) \Delta\varphi^2. \quad (5)$$

Similar expressions result for b and c , but as the higher angular variations of the second and third order temperature coefficients are negligible compared to the variations in the first order, they are not used further. It is sufficient to retain only the $\Delta\theta$ terms in b and c , as done by Bachmann:¹⁹

$$b = b_0 + (\partial b_0/\partial \theta) \cdot \Delta\theta , \quad (6)$$

$$c = c_0 + (\partial c_0/\partial \theta) \cdot \Delta\theta \quad (7)$$

Considering now only the first order temperature coefficient of frequency, $T_f^{(1)} = a$, the numerical values of the various derivatives are given in Table 2.²¹ Insertion of these values in (5) permits calculation of the change in $T_f^{(1)} = a$, due to angle changes:

$$\begin{aligned} \Delta a = (a - a_0) &= (\partial a_0/\partial \theta) \cdot \Delta\theta + (\partial a_0/\partial \varphi) \cdot \Delta\varphi + \\ &\quad \frac{1}{2}(\partial^2 a_0/\partial \theta^2) \cdot \Delta\theta^2 + (\partial^2 a_0/\partial \theta \partial \varphi) \cdot \Delta\theta \Delta\varphi + \\ &\quad \frac{1}{2}(\partial^2 a_0/\partial \varphi^2) \cdot \Delta\varphi^2. \end{aligned} \quad (8)$$

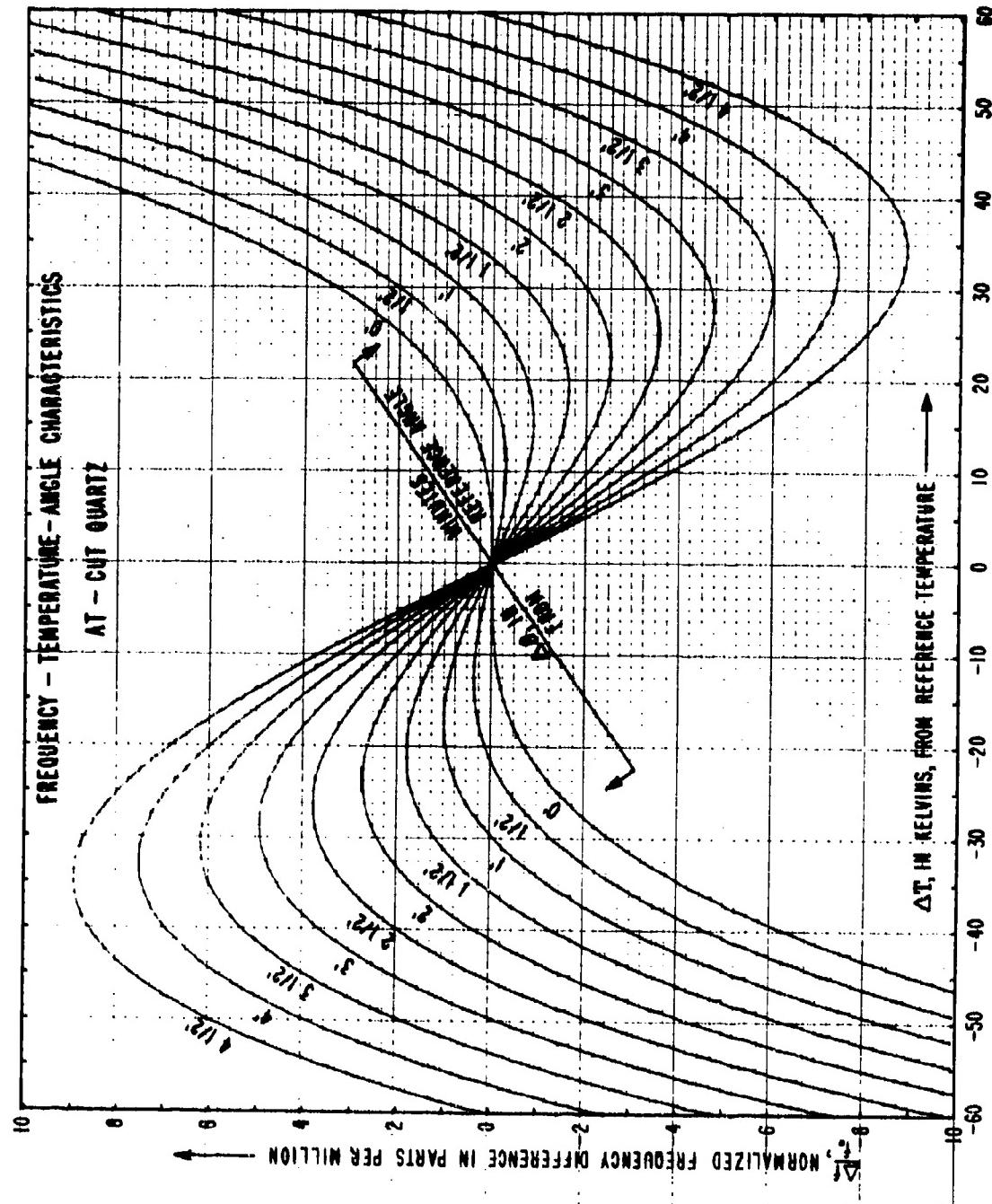


Figure 2. Frequency-temperature-angle Characteristics of AT-cut Quartz Resonators.

TABLE 2. ANGLE DERIVATIVES OF THE FIRST-ORDER TEMPERATURE COEFFICIENT OF FREQUENCY.

CUT	$\partial a_0 / \partial \theta$	$\partial a_0 / \partial \varphi$	$\partial^2 a_0 / \partial \theta^2$	$\partial^2 a_0 / \partial \theta \partial \varphi$	$\partial^2 a_0 / \partial \varphi^2$
	$10^6 / k, {}^\circ\theta$	$10^6 / k, {}^\circ\varphi$	$10^9 / k, ({}^\circ\theta)^2$	$10^9 / k, (\theta, {}^\circ\varphi)$	$10^9 / k, ({}^\circ\varphi)^2$
AT	-5.08	0	+0.96	0	-17.99

The numerical values in Table 2 reduce (8), with excellent accuracy, to

$$\Delta a = (\partial a_0 / \partial \theta) \cdot \Delta \theta + \frac{1}{2} (\partial^2 a_0 / \partial \varphi^2) \cdot \Delta \varphi^2 \quad (9)$$

so that, for constant Δa , a parabola describes the resulting curve. Setting $\Delta a = 0$ yields the curve of Figure 3, which describes the locus of constant first order temperature coefficient. For any other constant value of Δa , the parabola is simply shifted up or down along the $\Delta \theta$ axis.

For doubly rotated cuts in general, where the reference angle for φ is not zero, the curve will not be symmetric about $\Delta \varphi = 0$, owing to nonzero values for $\partial a_0 / \partial \varphi$.²¹

Figure 3 may be used in a variety of ways, but its chief virtue is that it allows one to determine the trade-off between variations in φ and in θ . It is seen that an offset in φ always leads to a curve of $\Delta f/f_0$ vs. T that appears to have a lowered θ value (cf. Figure 2). When a given application dictates error bounds on $\Delta \theta$, Figure 3 immediately determines the corresponding bounds on $\Delta \varphi$. This leads to consistency in specifications and cost savings because errors in $\Delta \varphi$ can be traded off against errors in $\Delta \theta$, and yields can be increased.

An application might lead to a specification of a maximum and a minimum value for $T_f^{(1)} = a$. Call these a_1 and a_2 . For each of these values, (9) determines a parabola of $\Delta \theta$ against $\Delta \varphi$. The region between the parabolas determines the combinations of $\Delta \theta$ and $\Delta \varphi$ angles such that the resulting plate temperature coefficient, $T_f^{(1)}$, will lie between a_1 and a_2 . Specification of this region rather than strict bounds on $\Delta \theta$ and $\Delta \varphi$ separately maximizes manufacturing yield.

Table 3 provides information in numerical form similar to that in Figure 3.

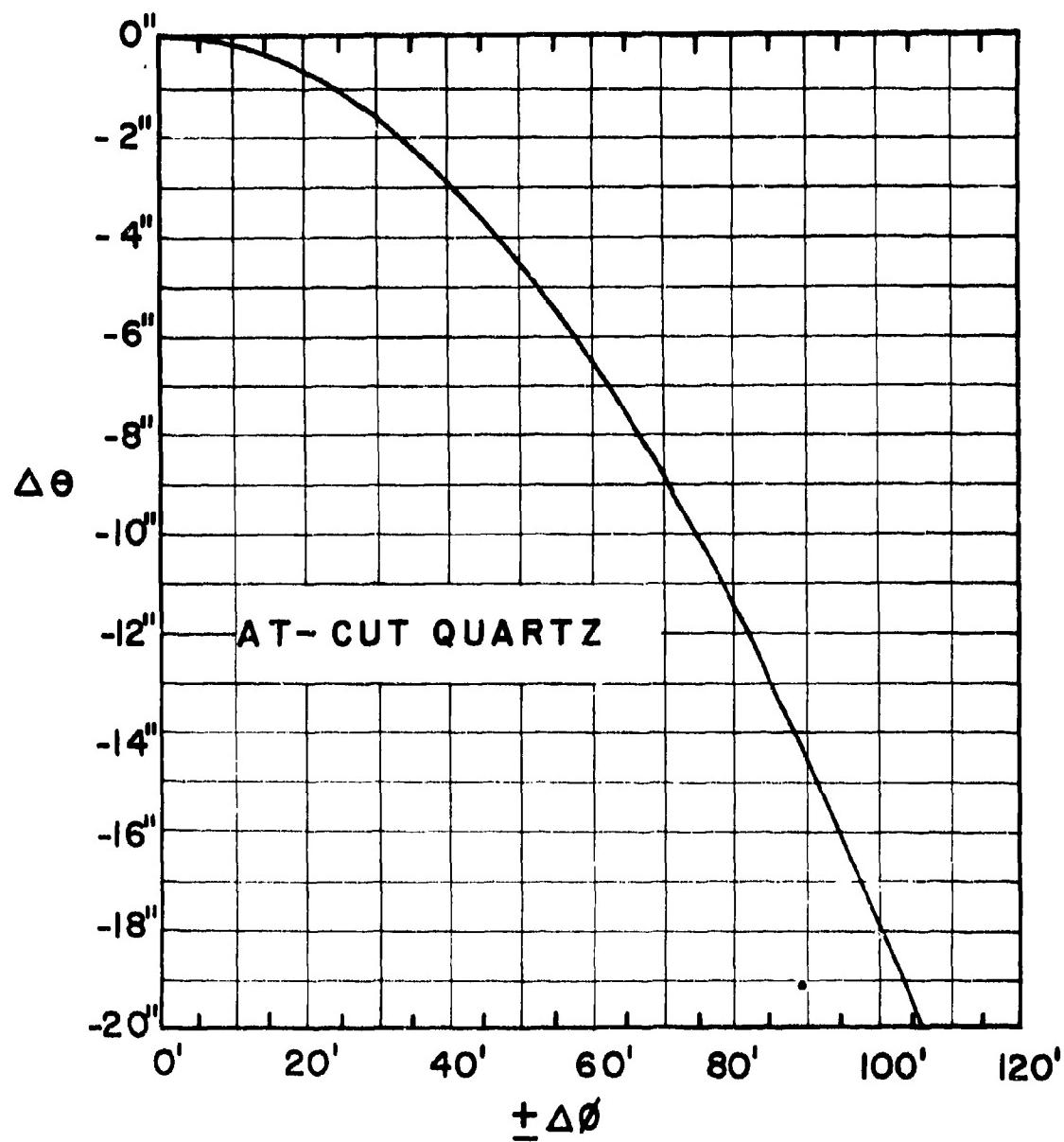


Figure 3. Locus of Constant First-order Temperature Coefficient Shift, as Function of Angle Deviations.

TABLE 3. ANGLE DEVIATIONS YIELDING ZERO
TEMPERATURE COEFFICIENT SHIFT

$\Delta \varphi$	$\Delta \theta$	$\Delta \varphi$	$\Delta \theta$
minutes	seconds	minutes	seconds
0	0	0	0
5	-0.04	23.8	-1
10	-0.18	33.6	-2
15	-0.40	41.2	-3
20	-0.71	47.5	-4
25	-1.11	53.1	-5
30	-1.59	58.2	-6
35	-2.17	62.9	-7
40	-2.83	67.2	-8
45	-3.59	71.3	-9
50	-4.43	75.1	-10
55	-5.36	78.8	-11
60	-6.38	82.3	-12
65	-7.49	85.7	-13
70	-8.68	88.9	-14
75	-9.97	92.0	-15
80	-11.3	95.0	-16
85	-12.8	98.0	-17
90	-14.4	100.8	-18
95	-16.0	103.6	-19
100	-17.7	106.2	-20

CONCLUSIONS

Stringent frequency control specifications require that the various parameters affecting the frequency-temperature behavior of crystal vibrators be identified, and their influences determined. This report provides the relations from which changes in the temperature coefficient due to changes in both orientation angles can be calculated. The results, in tabular and graphical forms, are provided as a practical aid in specifying tolerances.

REFERENCES

1. E. Hafner, "Crystal Resonators," IEEE Trans. Sonics Ultrason., Vol. SU-21, No. 4, Oct. 1974, pp. 220-237.
2. E. Hafner, "Frequency Control Aspects in Army Communications and Surveillance," Proc. 26th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1972, pp. 15-19.
3. F. E. Butterfield, "Frequency Control and Time Information in the NAVSTAR/Global Positioning System," Proc. 30th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1976, in press.
4. A. J. Van Dierendonck, "Time Requirements in the NAVSTAR/Global Positioning System (GPS)," Proc. 30th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1976, in press.
5. R. A. Maher, "Oscillator and Frequency Management Requirement for GPS User Equipments," Proc. 30th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1976, in press.
6. J. Moses, "NAVSTAR/Global Positioning System Oscillator Requirements for the GPS Manpack," Proc. 30th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1976, in press.
7. A. Ballato and T. J. Lukaszek, "Mass Effects on Crystal Resonators with Arbitrary Piezo-Coupling," Proc. 27th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1973, pp. 20-29.
8. A. Ballato and T. Lukaszek, "Mass-Loading of Thickness-Excited Crystal Resonators Having Arbitrary Piezo-Coupling," IEEE Trans. Sonics Ultrason., Vol. SU-21, No. 4, Oct. 1974, pp. 269-274.
9. A. Ballato and T. Lukaszek, "Mass-Loading Effects on Crystal Resonators Excited by Thickness Electric Fields," Technical Report ECOM-4270, US Army Electronics Command, Fort Monmouth, NJ, Oct. 1974, 80 pp.
10. A. Ballato, "Normalized Frequencies and Deviations as Function of Coupling, Mass-Loading and Harmonic: Numerical Results for Thickness Excitation," Technical Report ECOM-4354, US Army Electronics Command, Fort Monmouth, NJ, Sept. 1975, 164 pp.
11. A. Ballato, "Normalized Frequencies and Deviations as Function of Coupling, Mass-Loading and Harmonic: Numerical Results for Lateral Excitation," Technical Report ECOM-4404, US Army Electronics Command, Fort Monmouth, NJ, April 1976, 160 pp.
12. A. Ballato and T. Lukaszek, "Higher-Order Temperature Coefficients of Frequency of Mass-Loaded Piezoelectric Crystal Plates," Proc. 29th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, May 1975, pp. 10-25.

13. A. Ballato and T. Lukaszek, "Frequency-Temperature Coefficients of Mass-Loaded Plate Vibrators," Technical Report ECOM-4356, US Army Electronics Command, Fort Monmouth, NJ, Sept. 1975, 49 pp.
14. A. Ballato, "Alteration of Temperature Coefficients in Mass-Loaded Crystal Vibrators," Technical Report ECOM-4360, US Army Electronics Command, Fort Monmouth, NJ, Oct. 1975, 9 pp.
15. A. Ballato, "Apparent Orientation Shifts of Mass-Loaded Plate Vibrators," Proc. IEEE, Vol. 64, No. 9, Sept. 1976, in press.
16. R. Bechmann, "Influence of the Order of Overtone on the Temperature Coefficient of Frequency of AT-type Quartz Resonators," Proc. IRE, Vol. 43, No. 11, Nov. 1955, pp. 1667-1668.
17. M. Onoe, "Relationship between Temperature Behavior of Resonant and Antiresonant Frequencies and Electromechanical Coupling Factors of Piezoelectric Resonators," Proc IEEE, Vol. 57, No. 4, April 1969, pp. 702-703.
18. R. Bechmann, "Frequency-Temperature-Angle Characteristics of AT-type Resonators Made of Natural and Synthetic Quartz," Proc. IRE, Vol. 44, No. 11, Nov. 1956, pp. 1600-1607.
19. R. Bechmann, "Frequency-Temperature-Angle Characteristics of AT- and BT-type Quartz Oscillators in an Extended Temperature Range," Proc. IRE, Vol. 48, No. 8, Aug. 1960, p. 1494.
20. R. Bechmann, A. D. Ballato, and T. J. Lukaszek, "Higher-order Temperature Coefficients of the Elastic Stiffnesses and Compliances of Alpha-Quartz," Proc. IRE, Vol. 50, No. 8, August 1962, pp. 1812-1822, and Vol. 50, No. 12, Dec. 1962, p. 2451; USAELRDL Technical Report 2261, US Army Electronics R & D Lab., Fort Monmouth, NJ, Sept. 1963.
21. A. Ballato and G. J. Iafrate, "The Angular Dependence of Piezoelectric Plate Frequencies and Their Temperature Coefficients," Proc. 30th Annual Frequency Control Symp., US Army Electronics Command, Fort Monmouth, NJ, June 1976, in press.
22. "Standards on Piezoelectric Crystals, 1949." Proc. IRE, Vol. 37, No. 12, Dec. 1949, pp. 1378-1395. (IEEE Standard No. 176.)